

Noncommutative Q -balls

Youngjai Kiem^{a,*}, Chanju Kim^{b,†}, and Yoonbai Kim^{a,‡}

^a*BK21 Physics Research Division and Institute of Basic Science, Sungkyunkwan University,
Suwon 440-746, Korea*

^b*School of Physics, Korea Institute for Advanced Study,
Seoul 130-012, Korea*

Abstract

We obtain Q -ball solutions in noncommutative scalar field theory with a global $U(1)$ invariance. The Q -ball solutions are shown to be classically and quantum mechanically stable. We also find that “excited Q -ball” states exist for some class of scalar potentials, which are classically stable in the large noncommutativity limit.

Keywords: Noncommutative field theory, Q -balls, Nontopological solitons

*Email address: ykiem@newton.skku.ac.kr

†Email address: cjkim@kias.re.kr

‡Email address: yoonbai@skku.ac.kr

Apart from their string theoretic origin, noncommutative field theories possess a number of striking features that cannot be found for commutative counterparts [1]. A prime example is the existence of the classically stable, static soliton solution in noncommutative scalar field theories in more than $(1+1)$ -dimensions [2, 3]. The infinite number of derivatives appearing in the $*$ -products allows us to evade the arguments of Derrick and Hobart, which show that such solitons do not exist. Historically, one way to evade the arguments within the framework of ordinary field theories is to consider the stationary solutions in the presence of the continuous global symmetry, leading to Q -balls (or nontopological solitons) [4, 5, 6]. From the viewpoint of physical applications, Q -balls in supersymmetric theories have recently been suggested to be a possible candidate of dark matter [7]. In this paper, we show that it is possible to fuse these two lines of approaches. The analysis of Q -balls in noncommutative scalar field theories with a global $U(1)$ invariance can be performed by using the formalism of Gopakumar, Minwalla and Seiberg (GMS) [2], leading to noncommutative Q -balls.

According to the analysis of [2], the natural vantage point to study the GMS solitons is the large noncommutativity parameter θ limit. In this limit, the spatial kinetic terms are subleading, and exact nontopological GMS soliton solutions are obtained upon neglecting them. As we turn off the value of θ , GMS solitons cease to exist. In the ordinary Q -ball case ($\theta = 0$), the spatial kinetic terms can be neglected in the large Q limit (the thin-wall approximation). This is the limit where we have the analytic handle over Q -balls. As we turn on the value of θ and crank it up, the Q -balls continue to exist as nontopological solitons resembling the GMS solitons, as we will show in this paper. The simplification for the Q -ball physics coming from the large θ limit is that we can relax the large Q condition and still get the explicit form of the exact solutions.

While the noncommutative Q -ball solutions can be obtained by applying the techniques developed for the construction of GMS solitons, there are important differences even in the large θ limit. While the classically stable GMS solitons are constructed from the local minimum points of the potential, some noncommutative Q -balls can have classically stable “excited Q -ball states”, which also involve single local maximum point of the potential; we will derive the condition for the potentials for which such states exist. Furthermore, one can show that the noncommutative Q -balls are stable even quantum mechanically¹. On the other hand, GMS solitons are quantum mechanically metastable, since there is no conserved charge that can protect them from decaying into elementary quanta.

A simple (commutative) model field theory that possesses Q -balls is the $(2+1)$ -dimensional

¹Some subtleties related to the UV/IR mixing [8] that happens in the loop physics of noncommutative field theories will be discussed later in this paper. We show that our results should not be drastically modified up to one-loop order.

complex scalar theory with a global U(1) invariance

$$S = \int dt d^2x \left[\frac{1}{2} \partial_\mu \bar{\phi} \partial^\mu \phi - V(\bar{\phi}, \phi) \right], \quad (1)$$

where the scalar potential is given by

$$V(\bar{\phi}, \phi) = \frac{1}{2} m^2 \bar{\phi} \phi + \sum_{k=2}^N \frac{b_k}{(k!)^2} (\bar{\phi} \phi)^k. \quad (2)$$

The minimal potential that supports stable Q -balls is the one where b_2 and b_3 are nonzero and b_k 's with $k \geq 4$ vanish [5]. Noncommutative generalization of this minimal setup is our primary interest. Via the Weyl-Moyal correspondence, the replacement of products between the fields in (1) with the $*$ -products

$$(f * g)(x) = \exp \left(\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial z^\nu} \right) f(y) g(z) \Big|_{y=z=x} \quad (3)$$

should serve our purpose. For each term in the potential V beyond the quadratic order, however, there exist many inequivalent noncommutative generalizations; for example, depending on the ordering of fields, the six possible interaction vertices (modulo cyclic permutations)

$$\begin{aligned} & \bar{\phi} * \bar{\phi} * \bar{\phi} * \phi * \phi * \phi, \quad \bar{\phi} * \bar{\phi} * \phi * \bar{\phi} * \phi * \phi, \quad \bar{\phi} * \bar{\phi} * \phi * \phi * \bar{\phi} * \phi \\ & \bar{\phi} * \phi * \bar{\phi} * \bar{\phi} * \phi * \phi, \quad \bar{\phi} * \phi * \bar{\phi} * \phi * \bar{\phi} * \phi, \quad \bar{\phi} * \phi * \phi * \bar{\phi} * \bar{\phi} * \phi \end{aligned} \quad (4)$$

are all different, while all of them reduce to $(\bar{\phi} \phi)^3$ in the commutative limit. Among these terms that have the global U(1) invariance, the term

$$\bar{\phi} * \phi * \bar{\phi} * \phi * \bar{\phi} * \phi \quad (5)$$

can be singled out as the one that is also invariant under the local U(1) invariance. We also note that the potential term of the form $\bar{\phi} * \phi * \bar{\phi} * \phi$ was shown to be one-loop renormalizable in [9]. In this paper, we choose to only consider the interaction terms of the form

$$(\bar{\phi} * \phi)^k \equiv (\bar{\phi} * \phi) * (\bar{\phi} * \phi) * \cdots * (\bar{\phi} * \phi). \quad (6)$$

Implications of this choice will be addressed again later in this paper. We also note that we restrict our attention only to the spatially noncommutative field theories whose nonvanishing $\theta^{\mu\nu}$ components are $\theta^{xy} = -\theta^{yx} = \theta$. When considering Q -balls, this restriction significantly simplifies the analysis. The noncommutative version of the action (1) is thus given by

$$S = \int dt d^2x \left[\frac{1}{2} \partial_\mu \bar{\phi} * \partial^\mu \phi - V(\bar{\phi}, \phi) \right], \quad (7)$$

where the potential is

$$V(\bar{\phi}, \phi) = \frac{1}{2} m^2 \bar{\phi} * \phi + \sum_{k=2}^N \frac{b_k}{(k!)^2} (\bar{\phi} * \phi)^k . \quad (8)$$

Conserved Nöther current due to the global U(1) symmetry is

$$j_\mu = -\frac{i}{2} [\bar{\phi} * \partial_\mu \phi - (\partial_\mu \bar{\phi}) * \phi], \quad (9)$$

which yields the corresponding Nöther charge

$$\begin{aligned} Q &= \int d^2x j_0 \\ &= -i\pi\theta \operatorname{tr} [\bar{\phi} * \partial_0 \phi - (\partial_0 \bar{\phi}) * \phi]. \end{aligned} \quad (10)$$

When constructing the Nöther current (9), we utilize the property that the terms of the form (6) are invariant under the local U(1) symmetry. Inclusion of the other terms shown in (4) in the action (7) produces extra terms beyond the quadratic order in (9). In the second line of (10), we use the one-dimensional simple harmonic oscillator basis [2], which is known to be convenient for the construction of soliton solutions. The noncommuting spatial coordinates are related to the creation and annihilation operators via²

$$a = \frac{1}{\sqrt{2\theta}}(x + iy), \quad a^+ = \frac{1}{\sqrt{2\theta}}(x - iy) \quad \rightarrow \quad [a, a^+] = 1, \quad (11)$$

and

$$\int d^2x \quad \rightarrow \quad 2\pi\theta \operatorname{tr} . \quad (12)$$

The trace is taken with respect to this basis and the spatial derivatives inside the trace are understood as

$$\begin{aligned} \partial_+ \phi &= (\partial_x + i\partial_y)\phi = \sqrt{\frac{2}{\theta}} [a, \phi], \\ \partial_- \phi &= (\partial_x - i\partial_y)\phi = \sqrt{\frac{2}{\theta}} [\phi, a^+]. \end{aligned} \quad (13)$$

In terms of the harmonic oscillator basis, the energy can be written as

$$\begin{aligned} E &= \int d^2x \left\{ \frac{1}{2} |\partial_0 \phi|^2 + \frac{1}{2} |\partial_i \phi|^2 + V(\bar{\phi}, \phi) \right\} \\ &= 2\pi\theta \operatorname{tr} \left\{ \frac{1}{2} |\partial_0 \phi|^2 + \frac{1}{2\theta} [a, \phi][\bar{\phi}, a^+] + \frac{1}{2\theta} [a^+, \phi][\bar{\phi}, a] + V(\bar{\phi}, \phi) \right\} . \end{aligned} \quad (14)$$

²Throughout this paper, the coordinates x can either mean commuting coordinates or the operators defined on the noncommutative space.

Q -balls are stable field configurations with the minimum energy for a fixed, conserved $U(1)$ charge Q . In the commutative setup, the stationary form of the fields

$$\phi = e^{i\omega t}|\phi|(x, y) \quad , \quad \bar{\phi} = e^{-i\omega t}|\phi|(x, y) \quad (15)$$

satisfies the minimum energy condition [5]. The time derivative terms in (7) are quadratic and they are identical to those in the commutative case. Furthermore, the $*$ -products that appear in the potential involve only the spatial derivatives due to the restriction to the spatial noncommutativity. One can then show that the same stationary ansatz (15) provides us with the minimum energy configuration for a given charge Q in the noncommutative setup as well. For the stationary configurations, the charge (10) becomes

$$Q = \omega \int d^2x |\phi|^2 = 2\pi\theta \, \omega \, \text{tr} |\phi|^2. \quad (16)$$

Following [2], we analyze the system in the large θ limit. In this limit, the charge Q scales as θ , which implies that the time derivative term scales as θ just like the potential term. Meanwhile, the spatial kinetic terms are of the order of θ^0 and can be neglected in the sense of [2]. After neglecting the spatial derivative terms in the energy expression (14), it becomes

$$\begin{aligned} E &= \int d^2x \left\{ \frac{1}{2} |\partial_0 \phi|^2 + V(|\phi|) \right\} \\ &= \frac{Q^2}{2I} + 2\pi\theta \, \text{tr} V(|\phi|) , \end{aligned} \quad (17)$$

where

$$I = \int d^2x |\phi|^2 = 2\pi\theta \, \text{tr} |\phi|^2 . \quad (18)$$

Under the stationary ansatz (15), the potential depends only on $|\phi|(x, y)$. Corresponding equation of motion in the infinite θ -limit is given by

$$\ddot{\phi} = -\frac{\phi}{|\phi|} \frac{dV}{d|\phi|} \quad \rightarrow \quad \frac{dV}{d|\phi|} - \omega^2 |\phi| = 0 , \quad (19)$$

where we use the stationary ansatz (15). Equation (19) is an algebraic equation involving $*$ -products solved in [2]

$$\frac{d}{d|\phi|} V_{\text{eff}} = 0 , \quad (20)$$

where the effective potential V_{eff} is given by $V_{\text{eff}} = V - \frac{1}{2}\omega^2 |\phi|^2$. Therefore, the general (radially symmetric) solutions can be immediately written down as

$$|\phi| = \sum_n a_n |n\rangle \langle n|, \quad a_n \in \{\lambda_i\} , \quad (21)$$

where λ_i 's are the local extrema of V_{eff} . The corresponding energy and frequency ω are determined as

$$\begin{aligned} E &= \frac{Q^2}{2I} + 2\pi\theta \sum_n V(a_n) , \\ \omega &= \frac{Q}{I} , \end{aligned} \tag{22}$$

with $I = 2\pi\theta \sum_n a_n^2$. We note that λ_i 's are not extrema of V but of V_{eff} , which in turn depend on ω . The ω expression in (22) is thus implicit. We also mention that one has to go through the coherent basis analysis to obtain the position space form of the solutions. This procedure is identical to the one given in [2].

Given the solutions (21) satisfying the stationary ansatz (15), the next step is to identify physical solutions that are classically and quantum mechanically stable. We start from the classical stability analysis to see if there exist any negative modes for the small fluctuations around the classical solutions. In the case of GMS solitons [2], the classical stability of the soliton solutions requires that λ_i 's be local minimum points. As we will demonstrate shortly, however, there are some exceptional cases where this feature changes for noncommutative Q -balls. Following the analysis of [2], a simplifying condition for the classical stability analysis is that we consider the large θ limit neglecting the spatial kinetic terms. It can also be shown that it is sufficient to consider only radially symmetric fluctuations. Under the variation $\delta|\phi|$ of the solution (21) for a fixed Q

$$\delta|\phi| = \sum_n \delta_n |n\rangle \langle n| , \tag{23}$$

the energy changes as

$$\begin{aligned} \delta E &= \frac{1}{2} \int V''_{\text{eff}} (\delta|\phi|)^2 + \frac{2\omega^2}{I} \left(\int |\phi| \delta|\phi| \right)^2 + \mathcal{O}(\phi^3) \\ &\equiv \frac{1}{2} \sum_{m,n} \Delta_{mn} \delta_m \delta_n + \mathcal{O}(\phi^3) , \end{aligned} \tag{24}$$

where

$$\Delta_{mn} = 2\pi\theta V''_{\text{eff}}(a_n) \delta_{mn} + 4(2\pi\theta)^2 \frac{Q^2}{I^3} a_m a_n . \tag{25}$$

For the moment, we assume that all a_n 's are either zero or the local minimum point λ . The number of nonzero a_n 's is denoted as N . The $U(\infty)$ symmetry, which is exact in the large θ limit, allows us to set $a_n = \lambda$ for $n = 0, \dots, N-1$ and $a_n = 0$ for $n \geq N$, without loss of generality. The matrix Δ then becomes

$$\Delta = 4(2\pi\theta)^2 \frac{Q^2 \lambda^2}{I^3} \begin{pmatrix} A^{(1)} & 0 \\ 0 & A^{(0)} \end{pmatrix} , \tag{26}$$

where $A^{(1)}$ is an $N \times N$ matrix ($a, b = 0, 1, \dots, N-1$)

$$A_{ab}^{(1)} = 1 + \alpha \delta_{ab} , \quad (27)$$

with

$$\alpha = \frac{I^3}{8\pi\theta Q^2 \lambda^2} V_{\text{eff}}''(\lambda) , \quad (28)$$

and $A^{(0)}$ is proportional to the identity matrix ($c, d = N, N+1, \dots$)

$$A_{cd}^{(0)} = \frac{I^3}{8\pi\theta Q^2 \lambda^2} V_{\text{eff}}''(0) \delta_{cd} . \quad (29)$$

In the present case the quantity I is reduced to $I = 2\pi\theta N\lambda^2$. As far as the $\phi = 0$ is a stable local minimum, the eigenvalues of the matrix $A^{(0)}$ are positive. The N eigenvalues of $A^{(1)}$ are α ($(N-1)$ -times degenerate) and $\alpha + N$. Thus, as far as $\phi = \lambda$ is a stable local minimum, the solution

$$|\phi\rangle = \sum_{n=0}^{N-1} \lambda |n\rangle \langle n| \quad (30)$$

is classically stable.

We now consider the more general case when some of a_n 's are the local maximum point λ' with the condition $\lambda' < \lambda$. Two numbers N and M represent the numbers of λ and λ' appearing in a_n 's, respectively. The nontrivial part of the matrix Δ is then $(N+M) \times (N+M)$ submatrix Δ^{N+M} given by

$$\Delta^{N+M} = 4(2\pi\theta)^2 \frac{Q^2 \lambda^2}{I^3} \begin{pmatrix} A^{(1)} & A^{(2)} \\ A^{(2)} & A^{(3)} \end{pmatrix} , \quad (31)$$

where $A^{(1)}$, $A^{(2)}$ and $A^{(3)}$ matrices are given by ($a, b = 0, 1, \dots, N-1$ and $c, d = N, \dots, N+M-1$)

$$\begin{aligned} A_{ab}^{(1)} &= 1 + \alpha \delta_{ab} , \\ A_{ac}^{(2)} &= A_{ca}^{(2)} = \eta , \quad \eta = \frac{\lambda'}{\lambda} , \\ A_{cd}^{(3)} &= \eta^2 + \gamma \delta_{cd} , \quad \gamma = \frac{I^3}{8\pi\theta Q^2 \lambda^2} V_{\text{eff}}''(\lambda') < 0 , \end{aligned}$$

and $I = 2\pi\theta(N\lambda^2 + M\lambda'^2)$. We observe that the determinant of Δ^{N+M} is proportional to $\alpha^{N-1}\gamma^{M-1}$; Δ^{N+M} has the negative eigenvalue γ if $M > 1$. In other words, the solution is classically unstable when there are more than one local maximum points in a_n 's. The case of $M = 1$ is special and needs careful treatment; the eigenvalue equation for the Δ^{N+1} matrix reads

$$\det \left[\begin{pmatrix} A^{(1)} & A^{(2)} \\ A^{(2)} & A^{(3)} \end{pmatrix} - \mu \right] = (\alpha - \mu)^{N-1} [\mu^2 - (\alpha + N + \gamma + \eta^2)\mu + (\alpha + N)\gamma + \eta^2\alpha] = 0 . \quad (32)$$

We find that if the condition

$$-\frac{\eta^2\alpha}{\alpha+N} < \gamma < 0 \quad (33)$$

is satisfied, the eigenvalues of Δ^{N+1} are all positive. Contrary to naive expectations, there exists some possible parameter region where the solution is classically stable even when one of a_n 's is at a local *maximum* point of V_{eff} . We may call this solution an “excited Q -ball.” For the potential of the form $V = a(\bar{\phi} * \phi) + b(\bar{\phi} * \phi)^2 + c(\bar{\phi} * \phi)^3$ (renormalizable in the commutative setup in three dimensions), however, we emphasize that the condition (33) cannot be satisfied. For the effective potential V_{eff} with sufficiently ‘flattened’ shape near $\phi = \lambda'$, the condition can be met. We further note that the energy for this case is higher than that of the solutions (30) as will be shown later.

Quantum mechanically, Q -balls might dissociate into perturbative charged mesons. To ensure the stability against this decay channel, we have to compare the Q -ball energy to the threshold energy of the meson of the same charge at the tree level. If the condition

$$E \leq mQ \quad (34)$$

holds, such decay is energetically disfavored. Since it is natural to expect that the solutions with each $\{a_n\}$ corresponding to a local minimum will have the lower energy than other solutions, we first demonstrate (34) in this situation. We assume, without loss of generality, that the number of nonzero a_n 's is N and they have the local minimum value λ . The energy is then written as

$$E = 2\pi\theta \left\{ \frac{Q^2}{2(2\pi\theta)^2 N \lambda^2} + NV(\lambda) \right\} . \quad (35)$$

For a given Q , we have to choose the value of $N = N_{\min}$ such that the energy expression (35) is minimized. The appropriate value is

$$N_{\min} = \text{int} \left[\frac{Q}{2\pi\theta\lambda} \frac{1}{\sqrt{2V(\lambda)}} \right] + N_1 , \quad (36)$$

where $\text{int}[x]$ is the largest integer not larger than x and N_1 is zero or one depending on the situation. The minimum energy for sufficiently large N_{\min} is thus given by

$$E = Q \frac{\sqrt{2V(\lambda)}}{\lambda} , \quad (37)$$

and the value of ω in this case is

$$\omega = \frac{\sqrt{2V(\lambda)}}{\lambda} . \quad (38)$$

The expressions (37) and (38) are identical to the expressions derived for the commutative Q -balls when we identify λ as the height of the Q -ball. Moreover, the stability condition (34) translates to $\omega < m$ using (37) and (38), and this is equivalent to the condition

$$V(\lambda) < \frac{1}{2} V''(0)\lambda^2, \quad (39)$$

a result that is identical to the one in the commutative case. When the potential value at λ is smaller than the value of the quadratic part, Q -balls are stable against decay into charged mesons. In general, as far as we neglect the spatial kinetic terms, the expressions (37) and (38) are valid for any values of Q . We note that there are two limits where the spatial kinetic terms can be neglected; the large noncommutative parameter θ limit and the large Q limit (which is the case of the large Q thin-wall approximation limit in the commutative Q -ball setup). Although they are different limits, the above calculations are valid for both cases. In fact, the stability condition (34) can be generally proved by the following line of arguments, which is valid even when we include the spatial kinetic terms: From the equation of motion, we have

$$|\phi| * \partial_i \partial_i |\phi| + \omega^2 |\phi| * |\phi| = |\phi| * \frac{dV}{d|\phi|}. \quad (40)$$

Plugging (40) into the energy (14) leads to

$$\begin{aligned} E &= mQ - mQ + \frac{1}{2}\omega Q + \int d^2x V \\ &= mQ + 2\pi\theta \text{tr} \left\{ \omega(\omega - m) + V - \frac{1}{2}|\phi| \frac{dV}{d|\phi|} \right\}. \end{aligned} \quad (41)$$

We then use a Hölder-like inequality

$$\text{tr}|\phi|^2 \text{tr}|\phi|^6 \geq (\text{tr}|\phi|^4)^2, \quad (42)$$

which can be proved by diagonalizing $|\phi|$ and comparing both sides term by term. Following the arguments of [10], the quantity within in the curly bracket of (41) can be shown to be negative, which implies the condition (34), at least for the range of ω given in [10].

The solutions with one a_n corresponding to the local maximum point, the “excited Q -balls”, have higher energy than those considered above. Quantum mechanically, the former should decay into the latter. To demonstrate this aspect, we repeat the same calculation as the one given from (35) to (38). The value $N = N_{\min}$, which gives the minimum energy for the “excited Q -balls”, turns out to be

$$N_{\min} = \text{int} \left[\frac{Q}{2\pi\theta\lambda} \frac{1}{\sqrt{2V(\lambda)}} - \eta^2 \right] + N_1. \quad (43)$$

The corresponding energy for the sufficiently large N_{\min} is

$$E = \frac{Q}{\lambda} \sqrt{\frac{V(\lambda)}{2}} + 2\pi\theta[V(\lambda') - \eta^2 V(\lambda)] . \quad (44)$$

The energy of an “excited Q -ball” is larger than that of a Q -ball by the second term in (44). For the fixed charge Q , we can take the commutative limit where θ goes to zero while keeping the product θN_{\min} fixed (see (43)); we can still neglect the spatial kinetic term as long as the charge Q is large enough (Q -matter limit). Upon taking this limit the ratio, $-\eta^2\alpha/((\alpha+N)\gamma)$, goes like $\mathcal{O}(N^{-1})$ in (33), showing that the condition cannot be satisfied. The classically stable “excited Q -balls” do not exist in this limit. The classical stability of the “excited Q -balls” depends on the interplay between the noncommutativity and the existence of the conserved charge Q .

In the commutative field theory context, inclusion of the perturbative loop corrections in the quantum stability analysis does not qualitatively change the arguments given in the above. To be specific, upon including the one-loop corrections, the relation (34) is replaced by $E_Q + \delta E_Q \leq (m + \delta m)Q$ where δE_Q and δm are one-loop contributions. One can then consider the perturbative regime where the (renormalized) loop corrections are smaller than the tree contributions. In certain noncommutative field theories, however, the UV/IR mixing appears [8]; in noncommutative ϕ^4 theory in four dimensions, for example, the quadratic IR singularity present in the one-loop two-point 1PI amplitude drastically changes the low energy dispersion relations, casting doubts about the features observed at the classical level. In the present context with the global U(1)-respecting interaction terms (6) that also respect the local U(1) symmetry, the situation at the one-loop level is rather similar to that of commutative field theories.

In (2+1)-dimensions, the one-loop two-point 1PI amplitude has the linear UV divergence as can be seen by power counting, a potential source of the UV/IR mixing. The relevant interaction term is the $\bar{\phi} * \phi * \bar{\phi} * \phi$ term whose interaction vertex in the momentum space can be read off from [9]

$$\begin{aligned} & \frac{b_2}{4} \frac{1}{(2\pi)^3} \int dp_1 dp_2 dp_3 dp_4 \delta(p_1 + p_2 + p_3 + p_4) \\ & \times \cos\left(\frac{1}{2}(p_1\theta p_2 + p_3\theta p_4)\right) \bar{\phi}(p_1)\phi(p_2)\bar{\phi}(p_3)\phi(p_4) . \end{aligned} \quad (45)$$

One immediately notes that the θ -dependence in each possible one-loop two-point diagram drops out, demonstrating the lack of the UV/IR mixing. The commutative (renormalized) perturbative treatment still applies in our context as well. This analysis suggests that the qualitative features of Q -balls in consideration do not drastically change upon including the

one-loop correction. Had we included the term of the type $\bar{\phi} * \bar{\phi} * \phi * \phi$ in the action (7), the UV/IR mixing does occur. In addition, the current expression (9) gets modified and this can change the existence and properties of possible Q -ball solutions. At the higher loop level, the situation is more subtle. The two-loop corrections to the 1PI two-point amplitudes include the “sunset” diagram. By power counting the diagram has the logarithmic UV divergence in the commutative setup and the θ -dependence (determined by (45)) appears not to cancel out, suggesting the possible logarithmic UV/IR mixing. The detailed study of the two-loop corrections is thus clearly needed, but it is beyond the scope of this paper.

Acknowledgements

We are grateful to Jin-Ho Cho and Jaemo Park for useful discussions. This work is supported by No. 2000-1-11200-001-3 from the Basic Research Program of the Korea Science & Engineering Foundation.

References

- [1] For a review, see J.A. Harvey, hep-th/0102076 and references cited therein.
- [2] R. Gopakumar, S. Minwalla and A. Strominger, JHEP **0005**, 020 (2000), hep-th/0003160.
- [3] K. Dasgupta, S. Mukhi and G. Rajesh, JHEP **0006**, 022 (2000), hep-th/0005006; J. A. Harvey, P. Kraus, F. Larsen and E.J. Martinec, JHEP **0007**, 042 (2000), hep-th/0005031; D.J. Gross and N.A. Nekrasov, JHEP **0007**, 034 (2000), hep-th/0005204; A.P. Polychronakos, Phys. Lett. **B495**, 407 (2000), hep-th/0007043; D.P. Jatkar, G. Mandal and S.R. Wadia, JHEP **0009**, 018 (2000), hep-th/0007078; D. Bak, Phys. Lett. **B495**, 251 (2000), hep-th/0008204; D. Bak, K. Lee and J.-H. Park, hep-th/0011099; M. Li, hep-th/0011170.
- [4] R. Friedberg, T. D. Lee and A. Sirlin, Phys. Rev. **D13**, 2739 (1976).
- [5] S. Coleman, Nucl. Phys. **B262**, 263 (1985).
- [6] A. Kusenko, Phys. Lett. **B404**, 285 (1997), hep-th/9704073; M. Axenides, E. Floratos, S. Komineas and L. Perivolaropoulos, hep-ph/0101193.
- [7] A. Kusenko, Phys. Lett. **B405**, 108 (1997); A. Kusenko, V. Kuzmin, M. Shaposhnikov and P.G. Tinyakov, Phys. Rev. Lett. **80**, 3185 (1998).

- [8] S. Minwalla, M. V. Raamsdonk and N. Seiberg, hep-th/9912072; M. V. Raamsdonk and N. Seiberg, JHEP **0003**, 035 (2000).
- [9] I. Ya. Aref'eva, D. M. Belov and A. S. Koshelev, hep-th/0001215.
- [10] C. Kim, S. Kim and Y. Kim, Phys. Rev. **D47**, 5434 (1993).